

## **ERROR DIAGNOSIS OF PROBABILITY IN SCATTERING FROM POTENTIAL BARRIER OF USING RANDOM SIGNAL ANALYSIS**

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### **ABSTRACT**

Wireless networking is the branch of communication in which wireless channels are used for the transmission of the signals. There are various terminologies in the field of wireless networking which are taken from quantum physics. In fact quantum physics consists of the signals which are in wireless medium. There are many important theories which are given in the field of quantum mechanics which can be related with the concepts of wireless networks. In this paper we have tried to link the famous Heisenberg Uncertainty Principle and the Schrödinger wave equation with the wireless network in order to find the error in the position, momentum and energy of the network. A new concept is also discussed regarding the error in the probability in which an attempt is made to derive the equation of error with respect to the displacement. In order to derive these equations various concepts of mathematical modeling, Fourier series, Fourier transform, error diagnosis are discussed briefly.

**KEYWORDS:** Momentum, Wavelets, Duality, Uncertainty

### **1. INTRODUCTION**

Quantum Physics is the branch of science which highly deals with the electrons and other subatomic particles behaving as in the wave nature. Generally, scientists have two views that electron and other subatomic particles have two natures either wave or particle. Several theories were presented by the scientists to prove that electron follows wave nature. Generally in the field of wireless networking, waves are used for the transmission of the signal. The waves which are used here can be compared to the electron and the other sub-atomic particle waves as generated by the wave nature duality.

The paper includes the method through which the concepts of Schrödinger Equation can be modified to be used in wireless medium. The paper also includes the concepts of the probability in which the normal distribution of probability is used for the analysis of the theorems. We have tried to come up with a new concept in which the error in probability is introduced. The paper usually deals with the application layer of the wireless modeling. Probability of the energy density is studied between the nodes by considering cases of boundary conditions. Error in the probability will be derived by considering the Heisenberg and Schrödinger equation. Normally if we consider the normal form of distribution, usually constant step deviation leads to a constant probability but generally there is an error in the probability which can be linked through the error in the position, velocity or momentum of the particle.

The cases of Schrödinger equations which include the energy quantization, boundary conditions are taken from

[1]. The proof of the strong inequality was given by Kennard and Weyl [2]. Later Robertson [3] generalized the correlation for arbitrary observables  $A$  and  $B$ , and Dirichurn [4] presented the relation between Heisenberg's fluctuations. Generalized and précised form of Heisenberg's principle was given by Schrödinger [5, 6] and Robertson [7]. The Schrödinger's relation (3) can be expressed in a compact [6] and "very elegant form" through [9]. Ref. [8] explains about the role of noise in increasing the error. Ref [10] gives the information about the classical cases of quantum analogy used in handbooks on experimental physics. Ref. [11] gives the information regarding the matrix analogy for quantum physics. Definition of quantum mechanics is taken from [12]. There are two cases of Schrödinger equation as time independent and other as time dependent, we take ref. [13] in which the probability distribution of both the cases are studied.

The remainder of the paper is described in the following way: In the section II, the relation between the wireless networking and quantum physics is developed; approximation in the error in probability with respect to energy and position is defined and derived by considering the wave equations of Schrödinger wave equation. Results are explained in section III. Finally conclusion is in section IV.

## 2. EXTENSION OF PRINCIPLES

Extending the concepts of Heisenberg Uncertainty principle, we will have the uncertainty in the probability as there is a chance of having an uncertainty in the presence of connection between two nodes. As discussed the connection between two nodes depends on the energy density of the signals present between these signals. There we can say that the change of this probability will be dependent on the value of  $\Delta E$  as the value of  $p$  is related with the value of energy gradient. It has been described regarding the cutoff value which has been required for the connection between the two nodes.

$$\Delta p \cdot \Delta E \geq c'$$

Here  $c'$  is a constant whose value can be calculated from the practical experiments.

In the case of infinite wall potential and finite potential well, there are bound states which describes the presence of voltage barriers (continuous for fixed amount of distance and then change to remain continuous forever) which bounds the particle to be in a fixed arena due to its attractive force. But in the case of Potential Barriers, the voltage barrier is constant for all the displacement  $x > 0$  and zero for  $x < 0$ . It suggests that not all the fields are attractive, it is possible that the particle is free in the sense it is not confined to any sought of external field.

In order to analyze the concept of Scattering of Potential Barrier, take the case of Comet. Generally comet has the tendency to revolve around the sun. Comet starts its journey at infinity and ends by revolving the sun. Soon it reaches sun it takes a turn and again returns to infinity. If we consider when comet reaches sun it is having the displacement as zero then the potential voltage of the comet is constant throughout its return journey and it is independent on any type of external field exerted over it. Though, comets path is a bounded path which depends on the attraction of sun.

Scattering actually means the presence of attraction as well as repulsion in the same system. In the case of comet, first phase consists of the attraction field of the sun making it to come nearer to it and the second phase includes the repulsive field provided by the sun makes the comet to go again to infinity. Finally if there is attractive field then there is possibility of scattering as well as the particle to follow a bounded path, whereas if there is repulsive potential then scattering will occur. So it is possible for the particles to be confined in the limited space area and also satisfy the boundary

condition.

Before considering the potential barrier function with respect to displacement, there are certain assumptions which are taken:

$$\int_{-\infty}^{\infty} \psi(x) dx = 1$$

However this equation is valid for all the case in the probability distribution but it will be satisfied if

$$\psi(x) \rightarrow 0 \text{ as } x \rightarrow \pm\infty$$

Now Potential distribution of scattering condition is given as

$$V(x) = 0 \quad x < 0$$

$$= Vx > 0$$

Here barrier suggests the reflection of particle which occurs when there is movement in forward direction.

Now by the Schrödinger wave equation we have,

$$V^2 \psi(x) + \frac{8\pi^2 m}{h^2} (E - V) \psi(x) = 0 \text{ for } x > 0$$

$$V^2 \psi(x) + \frac{8\pi^2 m}{h^2} E \cdot \psi(x) = 0 \text{ for } x < 0$$

Here E is the total energy of the system.

To solve this equation consider first the analysis for  $x < 0$ , coefficient of  $\psi(x)$  be  $M^2$ . Therefore the equation changes to,

$$V^2 \psi(x) + k^2 \psi(x) = 0 \text{ for } x < 0$$

Therefore the value of k will be

$$k = \sqrt{8mE} * \frac{\pi}{h}$$

$$E = \frac{\left(\frac{M \cdot h}{\pi}\right)^2}{8m}$$

General solution of the above equation,

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

Generalized form of equation with time as a variable,

$$\psi(x, t) = \psi(x)e^{-i\omega t} = Ae^{i(kx - \omega t)} + Be^{-i(kx + \omega t)}$$

This equation represents the motion of waves for  $x < 0$ , firstly in the right direction and then in left direction.

Considering the second case for  $x > 0$ . Let the general equation be

$$V^2 \psi(x) - n^2 \psi(x) = 0 \text{ for } x > 0$$

$$N = \sqrt{8m(V_0 - E)} * \frac{\pi}{h}$$

$$E = \frac{\left(\frac{Mh}{\pi}\right)^2}{8m}$$

General solution for the equation:

$$\psi(x) = Ce^{nx} + De^{-nx}$$

Here C and D are constants

Now take boundary conditions in consideration, we have

$$\psi(x) \rightarrow 0 \text{ as } x \rightarrow \pm\infty$$

$\psi(x) \rightarrow \infty$  as  $x \rightarrow \infty$  if  $C \neq 0$ . So in order to satisfy the boundary condition  $C=0$ .

By taking the effects of both the variations in x, we have  $\psi(x)$  as

$$\begin{aligned} \psi(x) &= Ae^{i(kx-wt)} + Be^{-i(kx+wt)} \quad \text{for } x < 0 \\ &= De^{-nx} \quad \text{for } x > 0 \end{aligned}$$

To calculate the value of A, B and D,

Considering the discontinuity of the function at  $x=0$  describes the relation between the function defined for  $x>0$  and  $x<0$ . As it is a removable discontinuity, both the functions will have the tendency to have same values at  $x=0$ . Also the value of derivative of both the functions at  $x=0$  will be same. On applying the above analysis we get the following equations.

$$D = A + B$$

$$-n.D = ik(A - B)$$

On solving the above equation

$$B = \frac{(ik+n)}{(ik-n)} \cdot A$$

$$D = \frac{2ik}{(ik-n)} \cdot A$$

We have function as

$$\begin{aligned} \psi(x) &= Ae^{i(kx)} + \frac{(ik+n)}{(ik-n)} \cdot A \cdot e^{-i(kx)} \quad \text{for } x < 0 \\ &= \frac{2ik}{(ik-n)} \cdot A e^{-nx} \quad \text{for } x > 0 \end{aligned}$$

Considering the case for  $x<0$ , as  $\psi(x) \rightarrow \infty$  as  $x \rightarrow -\infty$ ; since the value of probability density function is not equal to zero as independent variable x tends to  $-\infty$ . So we neglect this condition for further error in probability diagnosis.

Generally case for  $x<0$ , deals with the reflection of the waves after coming in contact with potential barrier, the results shows that the input intensity is always equal to the output reflected intensity.

Considering the case for  $x > 0$ , we have wave function or probability density function as

$$P_x = |\psi(x)|^2 = \left| \frac{2ik}{(ik-n)} \cdot A e^{-nx} \right|^2$$

$$= \frac{4A.k^2}{k^2+n^2} e^{-2nx}$$

$$\sqrt{P_x} = \frac{2\sqrt{A}.k}{\sqrt{(k^2+n^2)}} e^{-nx}$$

Now if we consider the normal distribution of the probability then the probability function will be as following:

$$\psi(x)_{env} = \sqrt{P_x} = \sqrt{A_x} e^{-\frac{x-x_0}{4\sigma_x^2}}$$

On comparing the values of both the functions, we get the following result:

$$\frac{2\sqrt{A}.k}{\sqrt{(k^2+n^2)}} e^{-nx} = \sqrt{A_x} e^{-\frac{x-x_0}{4\sigma_x^2}}$$

$$\sqrt{A_x} = \frac{2\sqrt{A}.k}{\sqrt{(k^2+n^2)}}$$

Squaring both sides

$$A_x = \frac{4A.k^2}{k^2+n^2}$$

Second equation:

$$e^{-nx} = e^{-\frac{x-x_0}{4\sigma_x^2}}$$

Take natural log on both sides

$$\ln(e^{-nx}) = \ln\left(e^{-\frac{x-x_0}{4\sigma_x^2}}\right)$$

$$4n\sigma_x^2 = 1 - \frac{x_0}{x}$$

Differentiating the equation and finding the error function with respect to  $x$  and  $\sigma_x$ .

$$8n\sigma_x \Delta\sigma_x = \frac{x_0}{x^2} \Delta x$$

$$\Delta x = \frac{8n\sigma_x x^2}{x_0} \Delta\sigma_x$$

Now consider the probability density function in order to found the error in  $x$  and  $P$ .

$$P_x = \frac{4A.k^2}{k^2+n^2} e^{-2nx}$$

Differentiate  $P_x$  with respect to  $x$ .

$$\Delta P_x = -\frac{8A.k^2}{k^2+n^2} e^{-2nx} \cdot \Delta x$$

Now relation between errors in  $P$  with error in  $\sigma_x$

$$\Delta P_x = -\frac{8A.k^2}{k^2+n^2} \cdot \frac{8n\sigma_x x^2}{x_0} e^{-2nx} \Delta\sigma_x$$

$$\Delta P_x = -\frac{64.A.n\sigma_x k^2}{x_0.(k^2+n^2)} x^2 e^{-2nx} \Delta\sigma_x$$

Thus, we have defined a relation between  $\Delta P$  and  $\Delta\sigma_x$  that is the change in the value of step deviation with the error in probability, this equation could be used in order to find the relation between change in probability with the change in energy and change in momentum of the particle. Let a constant as  $Z_0$  having value

$$Z_0 = -\frac{64.A.n\sigma_x k^2}{x_0.(k^2+n^2)}$$

In order to this equation with Heisenberg Principle equations consider a variable  $Z$ , then value of  $Z$

$$Z = Z_0 \cdot x^2 e^{-2nx}$$

Therefore,

$$\Delta P_x = Z \cdot \Delta\sigma_x$$

Now, we can relate the error in p by others as by the given equations.

$$\Delta P \cdot \Delta p \geq Z/4\pi$$

$$\Delta P \cdot \Delta v \geq Z/4\pi m$$

Therefore from the above equations, the error in p is calculated in terms of  $\Delta x$ . If there is an increase in the value of  $\sigma_x$  and then the error in p is increased. Consider the case if  $\sigma_x \ll 0$ , then approximately we take the value of  $\sigma_x \cong 0$ . If we put the value of  $\sigma_x$  as zero in the above equation then following changes occurs.

$$\Delta P = -\lim_{\sigma_x \rightarrow 0} \frac{64.A.n.\sigma_x.k^2}{x_0.(k^2+n^2)} \cdot x^2 \cdot e^{-2nx} \Delta\sigma_x$$

$$\Delta P \cong 0$$

From the above equation it is clear that the error increases and decreases with the increase and decreases in value of  $\sigma_x$  respectively.

The error function is directly proportional to x which shows that with the increase in distance there is an increase in error.

## CONCLUSIONS

The paper consist a brief introduction about the error in the probability density function for scattering from Potential Barrier conditions of Schrödinger equation. From the given equations and the results, it is clear that the error in the probability is directly proportional to step deviation, due to which if the value of step deviation is low the error in probability is low. The desired value of step deviation should be low enough in order to have low error in probability. Also the function depends on x i.e. displacement (directly proportional), this shows with the increase of distance the error in probability is high.

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